

Exam P formula sheet

Probability · September 2026 Exam P syllabus

An independent study reference. Not an official SOA handout or a permitted personal exam reference. Definitions and conditions accompany each formula.

Events, counting, and conditional probability

Union and complement

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Subtract overlap once. The complement has probability $1 - P(A)$.

Total probability

$$P(A) = \sum_j P(A | B_j)P(B_j)$$

The events B_j must be disjoint and cover the sample space.

Permutations and combinations

$$P(n, r) = \frac{n!}{(n-r)!}, \quad \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Distinct objects, without replacement. Order matters for permutations.

Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Requires $P(B) > 0$. The denominator is the event already known.

Bayes' theorem

$$P(B_i | A) = \frac{P(A | B_i)P(B_i)}{\sum_j P(A | B_j)P(B_j)}$$

Requires positive total probability of A.

Independence

$$P(A \cap B) = P(A)P(B)$$

Disjoint events with positive probabilities are not independent.

Expectation, variance, and distribution functions

Expected value

$$E[g(X)] = \sum_x g(x)p_X(x) \quad \text{or} \quad \int g(x)f_X(x) dx$$

Sum or integrate over the support, with finite expectation when required.

Linear transformation

$$E[aX + b] = aE[X] + b; \quad \text{Var}(aX + b) = a^2 \text{Var}(X)$$

Adding a constant changes the mean but not the variance.

Continuous interval probability

$$P(a < X \leq b) = F_X(b) - F_X(a) = \int_a^b f_X(x) dx$$

For a continuous variable, including a single endpoint changes nothing.

Variance

$$\text{Var}(X) = E[X^2] - E[X]^2$$

Variance is nonnegative and has squared units.

CDF and survival

$$F_X(x) = P(X \leq x), \quad S_X(x) = 1 - F_X(x)$$

$S_X(x)$ means $P(X > x)$. Keep endpoint mass when X is discrete.

Discrete distributions

Binomial

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$k = 0, \dots, n$; independent trials. Mean np ; variance $np(1-p)$.

Negative binomial: trials through r th success

$$p(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$$

$k \geq r$; mean r/p ; variance $r(1-p)/p^2$. Failures equal $X-r$.

Poisson

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad E[X] = \text{Var}(X) = \lambda$$

$k = 0, 1, \dots$; λ is the expected count over the stated exposure.

Geometric: trials through first success

$$p(k) = (1-p)^{k-1} p, \quad E[X] = \frac{1}{p}$$

$k \geq 1$; variance $(1-p)/p^2$. For failures before success, subtract 1 from X .

Hypergeometric

$$p(k) = \frac{\binom{K}{k} \binom{N-K}{n-k}}{\binom{N}{n}}$$

n draws without replacement from N objects, K successes. Mean nK/N ; variance $n(K/N)(1-K/N)(N-n)/(N-1)$, $N > 1$.

Discrete uniform

$$E[X] = \frac{a+b}{2}; \quad \text{Var}(X) = \frac{(b-a+1)^2-1}{12}$$

Integers $a, \dots, b$ equally likely, each with probability $1/(b-a+1)$.

Continuous distributions

Uniform on (a,b)

$$f(x) = \frac{1}{b-a}; \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

$a < x < b$, $a < b$. Mean $(a+b)/2$; density is zero outside the interval.

Gamma: shape α , scale θ

$$f(x) = x^{\alpha-1} \frac{e^{-\frac{x}{\theta}}}{\Gamma(\alpha)\theta^\alpha}$$

$x > 0$; $\alpha, \theta > 0$. Mean $\alpha\theta$; variance $\alpha\theta^2$.

Normal

$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

X has mean μ and variance σ^2 , $\sigma > 0$. Standardize with $Z=(X-\mu)/\sigma$.

Exponential: scale θ

$$f(x) = \theta^{-1} e^{-\frac{x}{\theta}}; \quad S(x) = e^{-\frac{x}{\theta}}$$

$x > 0$, $\theta > 0$. Mean θ ; variance θ^2 . Rate $\lambda = 1/\theta$.

Beta on $(0,1)$

$$f(x) = x^{\alpha-1} \frac{(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

$\alpha, \beta > 0$; $B(\alpha,\beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha+\beta)$. Mean $\alpha/(\alpha+\beta)$; variance $\alpha\beta/[(\alpha+\beta)^2(\alpha+\beta+1)]$.

Discrete joint models and conditional moments

Marginal and conditional mass

$$p_X(x) = \sum_y p(x, y); \quad p_{X|Y}(x|y) = \frac{p(x, y)}{p_Y(y)}$$

Condition only on y with $p_Y(y) > 0$.

Covariance and correlation

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$$

Correlation divides covariance by $\sigma_X \sigma_Y$, requiring both standard deviations positive.

Total expectation

$$E[X] = \sum_y E[X|Y = y]p_Y(y)$$

Average the conditional means with the probabilities of their conditioning groups.

Joint expectation

$$E[g(X, Y)] = \sum_x \sum_y g(x, y)p(x, y)$$

Sum over allowed pairs. Impossible pairs carry zero mass.

Variance of a linear combination

$$\text{Var}(aX + bY) = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab \text{Cov}(X, Y)$$

Independence makes covariance zero; zero covariance alone need not imply independence.

Total variance

$$\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}(E[X|Y])$$

Within-group variation plus variation between conditional means.

Transformations and order statistics

CDF method

$$F_Y(y) = P(g(X) \leq y)$$

Find the transformed support first. Solve the inequality on every applicable branch.

Minimum and maximum

$$F_{\min}(x) = 1 - (1 - F(x))^n; \quad F_{\max}(x) = F(x)^n$$

For n independent, identically distributed observations with common CDF F.

kth order statistic density

$$f_{(k)}(x) = \frac{n!}{(k-1)!(n-k)!} F(x)^{k-1} (1 - F(x))^{n-k} f(x)$$

For iid continuous observations; $1 \leq k \leq n$.

One-to-one continuous transform

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|$$

For differentiable monotone g. Multiple inverse branches require adding their density contributions.

kth order statistic CDF

$$P(X_{(k)} \leq x) = \sum_{j=k}^n \binom{n}{j} F(x)^j (1 - F(x))^{n-j}$$

At least k of n iid observations must be at or below x.

Insurance payments and loss modifications

Deductible, coinsurance, and payment cap

$$Y = c \min(((1+r)X - d)_+, u)$$

X is the original loss, r inflation, d ordinary deductible, c insurer share. Here u caps the covered loss after the deductible, before coinsurance.

Positive-payment probability

$$P(Y > 0) = S_X\left(\frac{d}{1+r}\right)$$

Requires $c > 0$, $u > 0$, and $1+r > 0$.

Limited expected value

$$E[\min(X, u)] = \int_0^u S_X(x) dx$$

For nonnegative X and $u \geq 0$. A limit on loss differs from a limit on payment.

Exponential deductible

$$E[(X - d)_+] = \theta e^{-\frac{d}{\theta}}$$

Exponential scale θ and $d \geq 0$. Conditional excess has mean θ by memorylessness.

Expected payment per loss

$$E[Y] = c \int_d^{d+u} S_{(1+r)X}(x) dx$$

For the contract defined above, $X \geq 0$. Omit the upper cap by taking $u = \infty$.

Payment per payment

$$E[Y|Y > 0] = \frac{E[Y]}{P(Y > 0)}$$

Only when positive payments have positive probability. Do not confuse this with payment per loss.

Second moment of capped deductible payment

$$E[Y^2] = 2c^2 \int_0^u y S_{(1+r)X}(d + y) dy$$

Use $\text{Var}(Y) = E[Y^2] - E[Y]^2$. This uses the same contract and cap convention above.

Normal sums, quantiles, and approximation

Normal linear combination

$$\sum_i a_i X_i \sim N\left(\sum_i a_i \mu_i, \sum_i a_i^2 \sigma_i^2\right)$$

For independent normal variables. Add covariance terms when a jointly normal vector is dependent.

Normal quantile and symmetry

$$q_p = \mu + \sigma \Phi^{-1}(p); \quad \Phi(-z) = 1 - \Phi(z)$$

Φ is the standard normal CDF. Divide by standard deviation, not variance.

Central limit theorem

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \approx N(0, 1)$$

For iid variables with finite mean and nonzero finite variance, as n becomes large.

Normal approximation to binomial

$$P(X \leq k) \approx \Phi\left(\frac{k + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

For sufficiently large np and n(1-p). The 0.5 corrects the discrete boundary.

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