

Exam FM formula sheet

Financial Mathematics · August 2026 Exam FM syllabus

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Interest rates and accumulation

Effective interest and discount

$$v = \frac{1}{1+i}, \quad d = \frac{i}{1+i}, \quad i = \frac{d}{1-d}$$

All rates refer to the same period. Interest is measured against starting value; discount against ending value.

Nominal interest

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

i is annual effective; $i^{(m)}/m$ is the rate for one m th of a year.

Equivalent payment-period rate

$$i_h = (1 + i)^h - 1$$

h is the length of the payment period in years when i is annual effective.

Simple interest and discount

$$a(t) = 1 + it \quad \text{or} \quad a(t) = \frac{1}{1-dt}$$

These are separate models, not compound accumulation. Simple discount requires $1-dt > 0$.

Constant force of interest

$$\delta = \ln(1 + i), \quad a(t) = e^{\delta t}$$

Time t uses the same unit as the force δ .

Nominal discount

$$1 + i = \left(1 - \frac{d^{(m)}}{m}\right)^{-m}$$

Convert discount to accumulation before comparing with an interest quotation.

Variable force

$$a(t) = \exp\left(\int_0^t \delta(s) ds\right); \quad \delta(t) = \frac{a'(t)}{a(t)}$$

The accumulation function is positive and differentiable.

Changing effective rates

$$a(n) = \prod_{k=1}^n (1 + i_k)$$

Multiply accumulation factors; do not use an arithmetic average of rates.

Equations of value and cash-flow returns

Cash-flow value at time T

$$V_T = \sum_k C_k \frac{a(T)}{a(t_k)}$$

Use one sign convention and one comparison date.

Time-weighted return

$$1 + i_{TW} = \prod_k (1 + r_k)$$

Split at external cash flows; r_k is the investment return over each resulting interval.

NPV and yield

$$\text{NPV}(i) = \sum_k C_k (1 + i)^{-t_k}$$

An IRR is a root. Cash flows with multiple sign changes can have multiple roots.

Level annuities and payment timing

Annuity-immediate

$$a_{\overline{n}|} = \frac{1-v^n}{i}; \quad s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

Unit payments at 1,...,n. When $i = 0$, both factors equal n .

Deferred annuity

$$PV = v^m a_{\overline{n}|}$$

Unit payments at $m+1, \dots, m+n$; m is the deferral to the annuity valuation date.

mtlly annuity

$$a_{\overline{n}|}^{(m)} = \frac{1-v^n}{i^{(m)}}$$

Payments of $1/m$ at times $1/m, \dots, n$; i is annual effective and mn is an integer.

Annuity-due

$$a_{\text{due}, \overline{n}|} = (1+i)a_{\overline{n}|}$$

Payments at $0, \dots, n-1$. The accumulated due factor is also $(1+i)$ times the immediate factor.

Perpetuities

$$a_{\infty|} = \frac{1}{i}; \quad a_{\text{due}, \infty|} = \frac{1}{d}$$

Requires $i > 0$. Immediate starts at time 1; due starts now.

Continuous annuity

$$a_{\text{continuous}, \overline{n}|} = \frac{1-e^{-\delta n}}{\delta}$$

Unit payment rate over 0 to n with constant force δ . At $\delta = 0$ the value is n .

Increasing, decreasing, and geometric payments

Increasing annuity-immediate

$$(Ia)_{\overline{n}|} = \frac{a_{\text{due}, \overline{n}|} - nv^n}{i}$$

Payments $1, 2, \dots, n$ at times $1, 2, \dots, n$. For $i = 0$ use $n(n+1)/2$.

Arithmetic-gradient payments

$$PV = Pa_{\overline{n}|} + Q((Ia)_{\overline{n}|} - a_{\overline{n}|})$$

Payment at time t is $P+(t-1)Q$.

Growing perpetuity

$$PV = \frac{P_1}{i-g}$$

First payment P_1 at time 1; requires $i > g$ and $i, g > -1$.

Decreasing annuity-immediate

$$(Da)_{\overline{n}|} = (n+1)a_{\overline{n}|} - (Ia)_{\overline{n}|}$$

Payments $n, n-1, \dots, 1$.

Geometric payments

$$PV = P_1 \frac{1 - \left(\frac{1+g}{1+i}\right)^n}{i-g}$$

First payment P_1 at time 1, growth rate g . If $i = g$, $PV = nP_1/(1+i)$.

Loans and sinking funds

Level loan payment

$$P = \frac{L}{a_{\overline{n}|i}}$$

Loan L, n end-of-period payments, effective loan rate i per period.

Retrospective outstanding balance

$$B_k = L(1+i)^k - Ps_{\overline{k}|i}$$

Should equal the prospective balance under the same payment assumptions.

Principal progression

$$PR_k = Pv^{n-k+1}$$

For a level-payment loan; each next principal portion is (1+i) times the previous one.

Prospective outstanding balance

$$B_k = Pa_{\overline{n-k}|i}$$

Immediately after payment k. There are n-k remaining payments.

Interest and principal portions

$$I_k = iB_{k-1}; \quad PR_k = P - I_k$$

Interest uses the balance before payment k.

Sinking-fund deposit

$$D = \frac{L}{s_{\overline{n}|j}}; \quad \text{outlay} = Li + D$$

Interest-only loan L at rate i, accumulated fund repays L after n deposits earning rate j.

Bond prices and book values

Coupon bond price

$$P = Ca_{\overline{n}|j} + Rv_j^n$$

C is coupon per period, R redemption value, j yield per coupon period, $v_j = (1+j)^{-1}$.

Book-value recursion

$$B_{k+1} = B_k(1+j) - C$$

B_k is immediately after coupon k; this recurrence applies before final redemption.

Callable bond and guaranteed yield

$$P_{\max} = \min_{k \in \mathcal{C}} (Ca_{\overline{k}|j} + R_k v_j^k)$$

Test permitted call dates and maturity with their respective redemption amounts R_k .

Premium or discount

$$P - R = (C - Rj)a_{\overline{n}|j}$$

$P = R$ when $C = Rj$. Compare coupon amount with yield times redemption, not unlike rate conventions.

Price between coupon dates

$$P_{\text{dirty}} = B_k(1+j)^h$$

h is the elapsed fraction of a coupon period. Under straight-line accrued interest, clean price = dirty price - hC.

Duration, convexity, and yield sensitivity

PV weights and Macaulay duration

$$w_t = C_t \frac{v^t}{P}; \quad D_{\text{Mac}} = \sum_t t w_t$$

$P = \sum C_t v^t$ and $P \neq 0$. Time is measured in yield periods.

Convexity

$$\mathcal{C} = \frac{P''(i)}{P(i)} = \frac{\sum_t t(t+1)C_t(1+i)^{-t-2}}{P(i)}$$

This is modified convexity for changes in effective yield i .

Portfolio duration

$$D_{\text{portfolio}} = \sum_j \left(\frac{P_j}{\sum_k P_k} \right) D_j$$

Use present-value weights and matching definitions of duration.

Modified duration

$$D_{\text{mod}} = -\frac{P'(i)}{P(i)} = \frac{D_{\text{Mac}}}{1+i}$$

Cash flows are fixed and discounted with one effective yield i per period.

Price-change approximation

$$\frac{\Delta P}{P} \approx -D_{\text{mod}} \Delta i + \frac{1}{2} \mathcal{C} (\Delta i)^2$$

A local approximation with fixed cash flows. Use Δi as a decimal, in the same yield convention.

Asset–liability matching and immunization

Redington conditions

$$A(i_0) = L(i_0); \quad A'(i_0) = L'(i_0); \quad A''(i_0) > L''(i_0)$$

These give a local surplus minimum against small parallel changes in the common yield.

Full immunization for a single liability

$$\sum_j A_j v^{t_j} = L v^T; \quad \sum_j t_j A_j v^{t_j} = T L v^T$$

For a single liability at T , positive asset cash flows at dates on both sides of T , and a common flat yield. Match PV and duration.

Real return

$$1 + i_{\text{real}} = \frac{1 + i_{\text{nominal}}}{1 + \pi}$$

π is inflation for the same period. Nominal here means before inflation, not convertible mthly.

Two-asset duration weights

$$w = \frac{D_L - D_2}{D_1 - D_2}$$

Weights are PV fractions. Match PV as well; matching duration alone does not establish immunization.

Cash-flow matching

$$A_t = L_t \quad \text{at every payment date}$$

Exact matching by date removes the need to reinvest an unmatched interim cash flow.

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