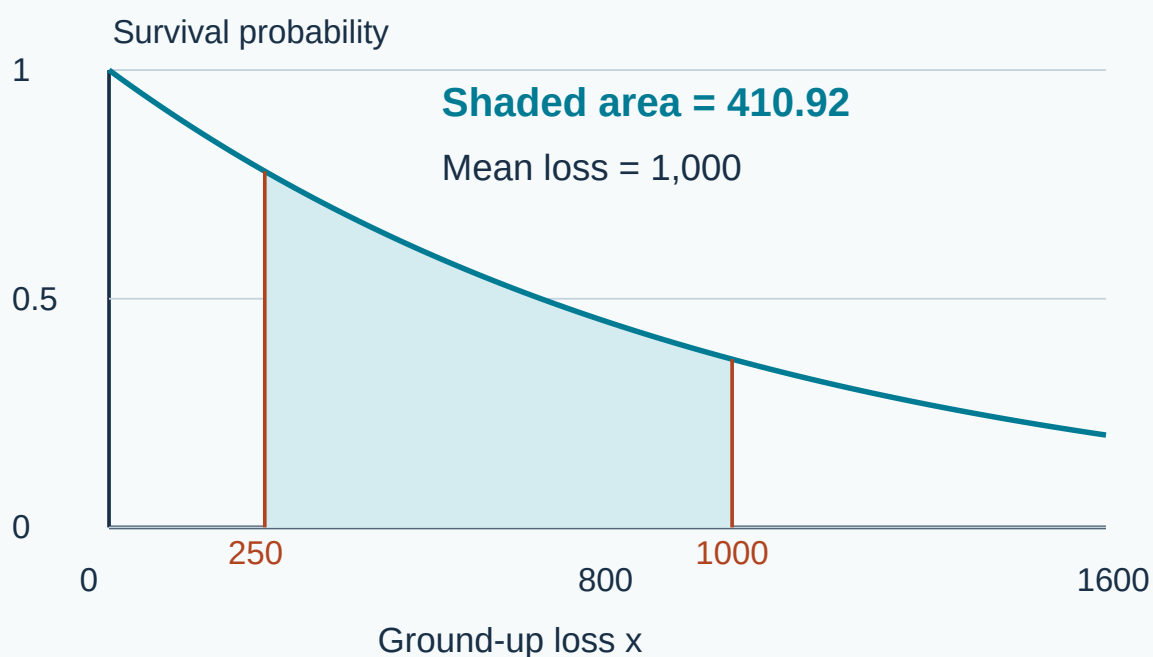


Tail integral: choose the loss thresholds

Loss survival and expected capped payment



Let X be exponential with mean 1,000 and let $Y = \min(\max(X - 250, 0), 750)$. The expected payment per loss is the area under the loss survival curve from 250 to 1,000.

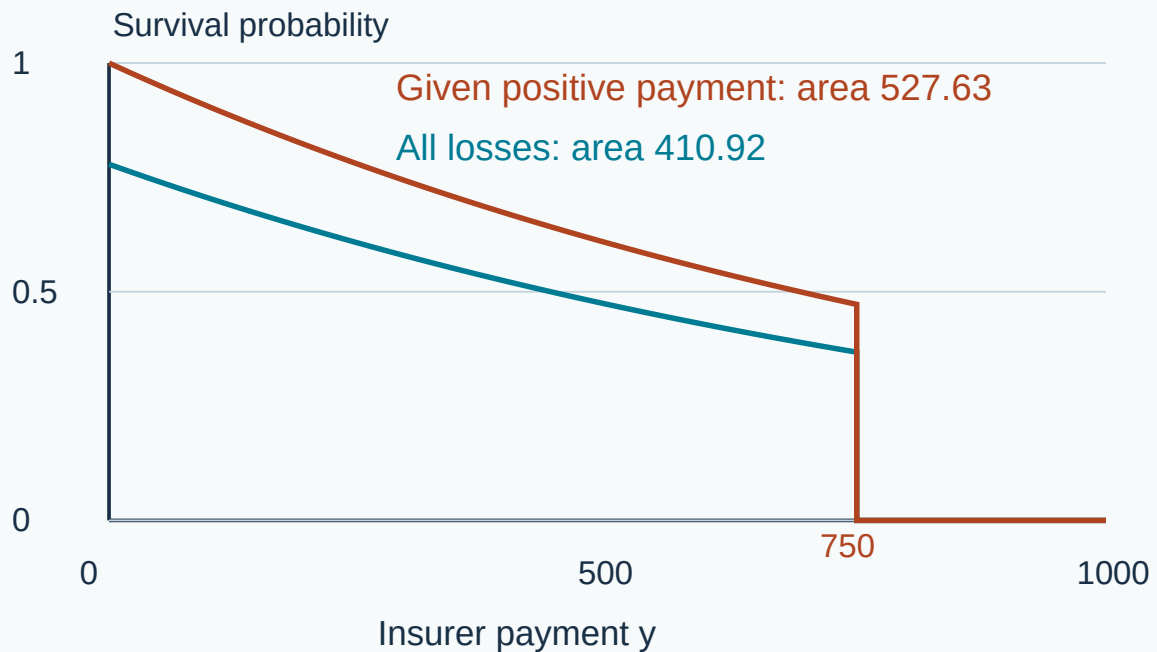
$E[Y] = 1,000 [\exp(-0.25) - \exp(-1)] = 410.921342$. The cap is a payment limit of 750; its upper loss threshold is deductible plus cap, or 1,000.

The one-integral tail identity assumes a nonnegative variable. Its value may be infinite. For a signed variable, separate the positive and negative tails.

Full proof, conditions and free problems: <https://actuaryproof.inshihub.com/exam-p/techniques/expected-value/>

Positive-payment conditioning changes the mean

Payment survival before and after conditioning



Probability of a positive payment = $\exp(-0.25) = 0.778801$. Mean per loss = 410.921342; mean per positive payment = 527.633447.

Conditioning on a positive payment divides the mean per loss by the probability of payment. It does not remove the payment cap.

At $y = 750$ the survival probability is zero. The nonzero left limit at that point is the atom at the cap; a single endpoint does not alter the integral.

With the deductible alone and no cap, mean per loss is 778.800783 and mean per positive payment is 1,000. Both are different from the capped example.

Free formula examples: <https://actuaryproof.inshihub.com/exam-p/formula-sheet/#worked-examples>

Rate conversion: compare one-year factors

Rate conversion at the same dates

Nominal annual 4%, compounded twice per year



$$\text{Constant annual force} = 2 \ln(1.02) = 0.0396052546$$

Annual effective interest = 4.04%; not the 4% quote

Nominal annual interest 4%, compounded semiannually, gives 2% per half-year. One unit becomes 1.0404 after one year. The equivalent constant annual force is 0.0396052546.

The equality is: $1 + \text{annual effective interest} = \exp(\text{annual force}) = 1 / (1 - \text{annual effective discount}) = (1 + \text{nominal interest} / \text{frequency})^{\text{frequency}}$.

The rate is constant and the periods are equal. A varying force needs an integral over time; a nominal discount quotation needs a different conversion.

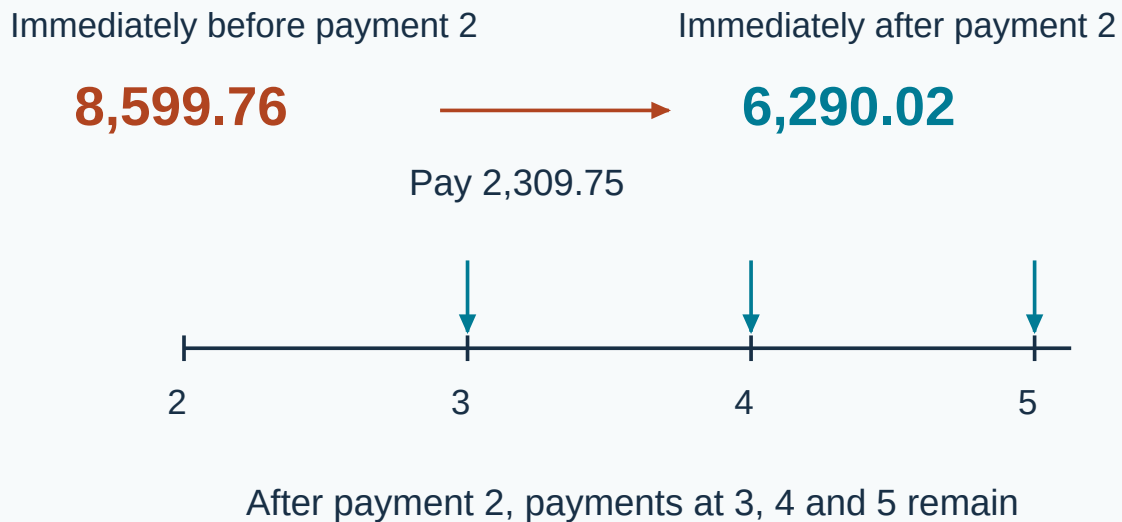
Change the rates in the free calculator:

<https://actuaryproof.inshihub.com/exam-fm/techniques/force-of-interest-and-rate-conversion/#interest-rate-calculator>

Complete FM sample 1 solution: <https://actuaryproof.inshihub.com/exam-fm/sample-1/>

Loan timing: do not count a payment twice

Loan balance around the second payment



A 10,000 loan is repaid with five annual payments at 5% effective annual interest. Unrounded payment = 2309.747981. Before payment two the balance is 8599.764620; after it the balance is 6290.016638.

Before payment two, the current payment and three future payments remain. After payment two, only the three future payments remain. Their present value is the after-payment balance.

Independent recurrence check: start at 10,000, multiply by 1.05, subtract the full payment, then repeat for payment two. Avoid rounding each step.

Existing worked example and free problems:

<https://actuaryproof.inshihub.com/exam-fm/techniques/loan-amortization-and-balances/>

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